

1. A family consisting of one mother, one father, two daughters and a son is taking a road trip in a sedan. The sedan has two front seats and three back seats. If one of the parents must drive and the two daughters refuse to sit next to each other, how many possible seating arrangements are there?

- (A) 28
- (B) 32
- (C) 48
- (D) 60
- (E) 120

As most of the combination problems this one can be solved in more than 1 way:

Sisters sit separately:

1. one of them is on the front seat (2 ways). Others (including second sister) can be arranged in: 2 (drivers seat)*3! (arrangements of three on the back seat)=12 ways. Total for this case: $2 \times 12 = 24$

Or

2. both by the window on the back seat (2 ways). Others can be arranged in: 2 (drivers seat)*2 (front seat)*1 (one left to sit between the sisters on the back seat)=4 ways. Total for this case=8.

Total=24+8=32.

Another way: Total number of arrangements-arrangements with sisters sitting together= $2 \times 4 \times 3! - 2 \times 2$ (sisters together)* $2 \times 2 \times 1$ (arrangement of others)=48-16=32

Answer: B.

2. What is the probability that a 3-digit positive integer picked at random will have one or more "7" in its digits?

- (A) 271/900
- (B) 27/100
- (C) 7/25
- (D) 1/9
- (E) 1/10

Total 3 digit numbers 900, 3 digit number with no 7 = $8 \times 9 \times 9 = 648$, $P(\text{at least one } 7) = 1 - P(\text{no } 7) = 1 - 648/900 = 252/900 = 7/25$

Answer: C.

3. A sphere is inscribed in a cube with an edge of 10. What is the shortest possible distance from one of the vertices of the cube to the surface of the sphere?

- (A) $10(\sqrt{3}-1)$
- (B) 5
- (C) $10(\sqrt{2}-1)$
- (D) $5(\sqrt{3}-1)$
- (E) $5(\sqrt{2}-1)$

Shortest distance=(diagonal of cube-diameter of sphere)/2= $\frac{10\sqrt{3}-10}{2} = 5(\sqrt{3}-1)$

Answer: D.

4. A contractor estimated that his 10-man crew could complete the construction in 110 days if there was no rain. (Assume the crew does not work on any rainy day and rain is the only factor that can deter the crew from working). However, on the 61-st day, after 5 days of rain, he hired 6 more people and finished the project early. If the job was done in 100 days, how many days after day 60 had rain?

- (A) 4
- (B) 5
- (C) 6
- (D) 7
- (E) 8

This one was solved incorrectly:

Days to finish the job for 10 people 110 days.

On the 61-st day, after 5 days of rain --> 5 days was rain, 55 days they worked, thus completed 1/2 of the job, 1/2 is left (55 days of work for 10 people).

Then 6 more people was hired --> speed of construction increased by 1.6, days needed to finish $55/1.6=34.375$, BUT after they were hired job was done in $100-60=40$ days --> so 5 days rained. They needed MORE than 34 days to finish the job, so if it rained for 6 days they wouldn't be able to finish the job in 100(40) days.

Answer: B.

5. If s and t are positive integer such that $s/t=64.12$, which of the following could be the remainder when s is divided by t ?

- (A) 2
- (B) 4
- (C) 8
- (D) 20
- (E) 45

s divided by t yields the remainder of r can always be expressed as: $\frac{s}{t} = q + \frac{r}{t}$ (which is the same as $s = qt + r$), where q is the quotient and r is the remainder.

Given that $\frac{s}{t} = 64.12 = 64\frac{12}{100} = 64\frac{3}{25} = 64 + \frac{3}{25}$, so according to the above $\frac{r}{t} = \frac{3}{25}$, which means that r must be a multiple of 3. Only option E offers answer which is a multiple of 3

Answer: E.

6. A committee of 6 is chosen from 8 men and 5 women so as to contain at least 2 men and 3 women. How many different committees could be formed if two of the men refuse to serve together?

- (A) 3510
- (B) 2620
- (C) 1404
- (D) 700
- (E) 635

Committee can have either: 2 men and 4 women OR 3 men and 3 women (to meet the condition of at least 2 men and 3 women).

Ways to chose 6 members committee without restriction (two men refuse to server together): $C_8^2 * C_5^4 + C_8^3 * C_5^3 = 700$

Ways to chose 6 members committee with two particular men serve

together: $C_2^2 * C_5^4 + C_{22}^2 * C_6^1 * C_5^3 = 5 + 60 = 65$

$700 - 65 = 635$

Answer: E.

7. If x is positive, which of the following could be the correct ordering of $1/x$, $2x$ and x^2 ?

- I. $x^2 < 2x < 1/x$
- II. $x^2 < 1/x < 2x$
- III. $2x < x^2 < 1/x$

- (A) None
- (B) I only
- (C) III only
- (D) I and II only
- (E) I II and III

First note that we are asked "which of the following COULD be the correct ordering" not MUST be.

Basically we should determine relationship between x , $\frac{1}{x}$ and x^2 in three areas: $0 < 1 < 2 < x$

$x > 2$

$$1 < x < 2$$

$$0 < x < 1$$

When $x > 2 \rightarrow x^2$ is the greatest and no option is offering this, so we know that $x < 2$.

If $1 < x < 2 \rightarrow 2x$ is greatest then comes x^2 and no option is offering this.

So, we are left with $0 < x < 1$:

In this case x^2 is least value, so we are left with:

I. $x^2 < 2x < \frac{1}{x} \rightarrow$ can $2x < \frac{1}{x}$? Can $\frac{2x^2-1}{x} < 0$, the expression $2x^2-1$ can be negative or positive for $0 < x < 1$.
(You can check it either algebraically or by picking numbers)

II. $x^2 < \frac{1}{x} < 2x \rightarrow$ can $\frac{1}{x} < 2x$? The same here $\frac{2x^2-1}{x} > 0$, the expression $2x^2-1$ can be negative or positive for $0 < x < 1$. (You can check it either algebraically or by picking numbers)

Answer: D.

8. In the xy plane, Line k has a positive slope and x-intercept 4. If the area of the triangle formed by line k and the two axes is 12, What is the y-intercept of line K?

- (A) 3
- (B) 6
- (C) -3
- (D) -6
- (E) -4

Positive slope, positive (4) x-intercept \rightarrow negative y-intercept. $\rightarrow \frac{1}{2} \cdot 4 \cdot |y| = 12 \rightarrow |y| = 6. \rightarrow y = -6$

Answer: D

9. Of the applicants passes a certain test, 15 applied to both college X and Y. If 20 % of the applicants who applied college X and 25% of the applicants who applied college Y applied both college X and Y, how many applicants applied only college X or college Y?

- (A) 135
- (B) 120
- (C) 115
- (D) 105
- (E) 90

$20\%X = X \& Y = 15 \rightarrow X = 75 \rightarrow$ Only $X = 75 - 15 = 60$

$25\%Y = X \& Y = 15 \rightarrow Y = 60 \rightarrow$ Only $Y = 60 - 15 = 45$

Only X or Y = $60 + 45 = 105$

Answer: D.

10. What is the lowest positive integer that is divisible by each of the integers 1 through 7, inclusive.

- (A) 420
- (B) 840
- (C) 1260
- (D) 2520
- (E) 5040

The integer should be divisible by: 2, 3, 4(=2^2), 5, 6(=2*3), and 7. LCM = $2^2 \cdot 3 \cdot 5 \cdot 7 = 420$

Answer: A.